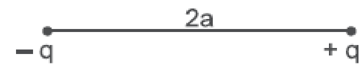
ELECTRIC DIPOLE

An electric dipole a pair of two equal and opposite charges placed a small distance apart.

Electric dipole moment

The electric dipole moment of a dipole is defined as the product of either of the charges and the distance between them.



Mathematically $\vec{p} = q \times \vec{2a}$

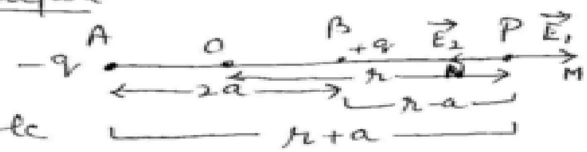
It is a vector directed from negative to the positive charge. Its SI unit is Cm (coulomb-meter).

In nature every polar molecule is an electric dipole.

DIPOLE FIELD AT A POINT ON AXIAL LINE

Electric Field intensity at a point ~~due~~ on the axial line of an electric dipole

Consider a point P at a distance 'r' from the mid point O of an electric dipole $(\pm q, 2a)$ as shown.



Field intensity at P due to +q charge $= \vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{q}{(r-a)^2}$ along $\vec{PM}$

" " " P " " −q " $= \vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{q}{(r+a)^2}$ along $\vec{PN}$

Net field at $P = \vec{E} = (E_1 - E_2)$ along $\vec{PM}$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] \text{ along } \vec{PM}$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\epsilon_0}\cdot\frac{4ar}{(r^2-a^2)^2} = \frac{2(q.2a).r}{4\pi\epsilon_0(r^2-a^2)^2} \text{ along } \vec{PM}$$

$$= \frac{2\vec{p}\,r}{4\pi\epsilon_0(r^2-a^2)^2}$$ [$\because \vec{PM}$ is along $\vec{p}$ i.e. −ve to +ve charge

For a short dipole ; $(r^2-a^2)^2 \simeq r^4$ [Neglect a]

$$\therefore \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{2\vec{p}}{r^3} \Rightarrow E \propto p \text{ and } E \propto \frac{1}{r^3}.$$

Note :- If distance is doubled ; the field intensity becomes one - eighth .

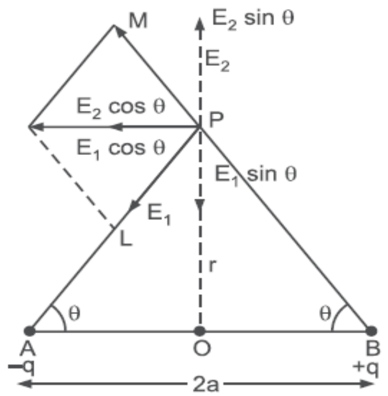
Electric field intensity at a point on equitorial line of an electric dipole

Consider a point P at a distance 'r' from the mid-point 'O' on the equitorial line of an electric dipole ($\pm q$, $2a$) as shown.

Electric field intensity at P due to charge at A

$$= \overrightarrow{E_{PA}} = \vec{E_1} = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} \text{ along } \overrightarrow{PA}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2} \text{ along } \overrightarrow{PA}$$



Similarly $$= \overrightarrow{E_{PB}} = \vec{E_2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)} \text{ along } \overrightarrow{PM}$$

Resolving $\vec{E_1}$ and $\vec{E_2}$ along $\vec{r}$ and $\perp \vec{r}$; the components along $\vec{r}$ cancel out whereas components $\perp \vec{r}$ add up.

$\therefore$ Net field $\vec{E} = E_1\cos\theta + E_2\cos\theta$

$$= 2E_1\cos\theta \qquad \left[\because \left|\vec{E_1}\right| = \left|\vec{E_2}\right|\right]$$

$$= 2\cdot\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r^2+a^2}\right)\cdot\frac{a}{(r^2+a^2)^{1/2}}$$

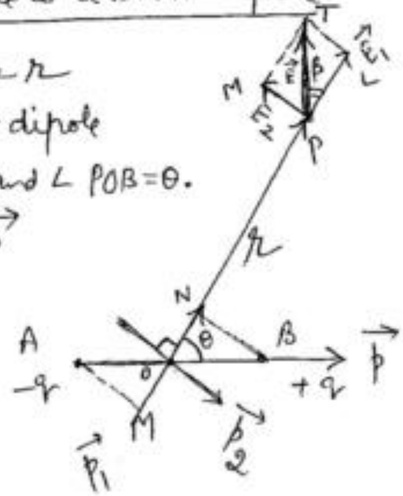
$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}} \text{ along } \overrightarrow{PX}$$

$$\therefore \quad \vec{E} = -\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{\vec{p}}{(r^2+a^2)^{3/2}}$$ [– ve sign $\vec{E}$ indicates opposite to $\vec{p}$]

For a short dipole; 'a' can be neglected as compared to 'r'.

$$\therefore \vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{\vec{p}}{r^3}$$

Field Intensity at any point due to a short dipole



Let P be a point at a distance r from mid-point 'O' of a short dipole with electric dipole moment $\vec{p}$ and $\angle POB = \theta$.

Resolving dipole moment $\vec{p}$ along $\vec{r}$ and normal to $\vec{r}$;

the components are

$\vec{p}_1 = p\cos\theta$ along $\vec{r}$

& $p_2 = p\sin\theta \perp \vec{r}$.

Field intensity at P due to $\vec{p}_1$ is given by

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}\frac{2p_1}{r^3} = \frac{1}{4\pi\epsilon_0}\frac{2p\cos\theta}{r^3}$$ along PL [As P lies on axial line of $\vec{p}_1$]

Similarly P being on equitorial line of $\vec{p}_2$; the field intensity due to it at P is

$$\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{p_2}{r^3} = \frac{1}{4\pi\epsilon_0}\frac{p\sin\theta}{r^3}$$ along $\vec{PM}$ [Opposite to $\vec{p}_2$]

Also $\vec{E}_2 \perp \vec{E}_1$

$\therefore$ Net field at P $= \vec{E} = \sqrt{E_1^2 + E_2^2} = \frac{p}{4\pi\epsilon_0 r^3}\left[\sin^2\theta + 4\cos^2\theta\right]$

$$= \frac{p}{4\pi\epsilon_0 r^3}\left[1 + 3\cancel{\sin}^2\theta\right]$$ along $\vec{PT}$

Let $\vec{E}$ make an angle β with $\vec{r}$.

Then $\tan\beta = \frac{LT}{PL} = \frac{E_2}{E_1} = \frac{1}{2}\tan\theta$.

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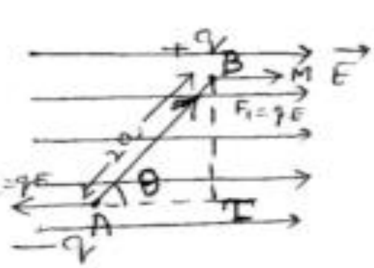
Dipole ~~in~~ in a uniform electric field



Consider an electric dipole ($\pm q, 2a$) in a uniform electric field of intensity $\vec{E}$. Let θ be the angle between $\vec{p}$ and $\vec{E}$.

Force on $-q$ charge due to the field $= \vec{F_1} = (-q)\vec{E} = qE$ along $\vec{AL}$

" " $+q$ " " " " " $= \vec{F_2} = (+q)\vec{E} = qE$ along $\vec{BM}$.

$\therefore$ Net force on dipole in uniform field = zero.

The forces being equal, unlike and parallel with different lines of action form a couple

Moment of the couple $= \tau =$ Either force $\times$ $\perp$lar distance between forces

or $\tau = (qE) \cdot BT = (qE)\, AB \sin\theta = (q \cdot E)(2a \sin\theta)$

$= (q\, 2a)\, E \sin\theta = pE \sin\theta.$

In vector form $\vec{\tau} = \vec{p} \times \vec{E}$.

The direction of torque is given by right-hand (cross product) rule applied to $\vec{p} \times \vec{E}$.

Condition for maximum Torque – $\theta = 90^\circ$

$\therefore \tau_{max} = pE \sin 90^\circ$

$= pE$

For minimum Torque; $\theta = 0^\circ$ or 180°

$\tau_{min} = 0$ when the dipole is parallel or antiparallel to the field.

P S : Please note that the above topics are very important.

After learning these basics, please try a few simple numerical problems from previous year CBSE papers. You will find these problems in every book.

The numerical problems asked in the board exams are simple applications of the facts and relation learnt. Try them with confidence.

You just need to take a step forward.

A comprehensive exercise on Electrostatics will also be posted shortly.