CURRENT ELECTRICITY -2

RESISTANCE,RESISTIVITY & OHM'S LAW

Current Electricity -2

Factors affecting resistance :- The resistance R of a metallic conductor is (i) directly proportional to length 'l'
(ii) inversely to its area of cross-section 'A'

i.e $R \propto l$; $R \propto \frac{1}{A} \Rightarrow R = \rho \frac{l}{A}$ where ρ is a constant called resistivity of the material of the conductor. The value of ρ is independent of shape and size but depends on the temperature in addition to the nature of the material. For $l = 1m$; $A = 1\ sq.m$; $\rho = R$

Hence resistivity of the material of a conductor is numerically equal to the resistance of a 1m long conductor with a uniform area of cross-section 1 sq.m. Its SI unit is Ω-m. Its value varies from very low for metals (e.g. $1.6 \times 10^{-8}\ \Omega m$) for Silver to very high (e.g. 10^{13} to 10^{16}) for rubber.

Ohm's Law [Recall $I = \frac{ne^2A\tau}{m} E$ for a conductor (Current Elect-I)]

For a conductor $I = \frac{ne^2A\tau}{m} E = \frac{ne^2A\tau}{m} \frac{V}{l}$ $\left[As\ E = \frac{V}{l}\right]$

$\therefore \frac{V}{I} = \left(\frac{m}{e^2 n \tau}\right) \frac{l}{A}$. For a conductor of given length and area of cross-section, all the terms on RHS are constant at a given temperature (τ varies with temperature decreasing with increasing temperature)

Hence $\frac{V}{I}$ = constant which is Ohm's law.

Hence Ohm's law can be stated as under:

" For a conductor at a fixed temperature, the potential difference applied across its ends is directly proportional to the current passing through it.

VARIATION OF RESISTANCE WITH TEMPERATURE

Expression for resistance (We have $R = \frac{V}{I}$) defined as the measure of ability of a conductor to oppose the flow of current through itself. [Scalar with SI units ohm (Ω)].

Using $\frac{V}{I} = \frac{ml}{e^2 n \tau A}$ and $R = \frac{V}{I}$, we get

$R = \frac{ml}{e^2 n \tau A}$ which is the required expression.

Expression for resistivity Comparing $R = \rho \frac{l}{A}$ and $R = \left(\frac{m}{e^2 n \tau}\right) \frac{l}{A}$ we get

$$\boxed{\rho = \frac{m}{e^2 n \tau}}$$

Effect of temp. on resistance/resistivity

(i) **Conductors** For a conductor; the resistance is caused by the collisions between the electrons as they ~~drift~~ are in constant random thermal motion. At higher temperature; the random motion increases resulting in more frequent collisions lesser free time τ. Hence the resistance and resistivity of a conductor increase with rise in temperature. [for graphical representation of ρ vs T see NCERT book]

Mathematically $R_t = R_0[1 + \alpha t]$ where R_t is resistance at $t°C$; R_0 at $0°C$ and α is the temperature co-efficient of resistance. $\left[\alpha = \frac{R_t - R_0}{R_0 . t}\right]$ defined as the increase in resistance per unit resistance per °C rise in temperature. (or per kelvin)

SI unit of α is K^{-1}.

ELECTRIC ENERGY AND POWER

(ii) In a semiconductor, as temperature increases, the bonds between atoms break up resulting in a sharp increase in the number of free charge carriers. (i.e. electrons and holes) increases. Hence the resistivity/resistance of semiconductors decreases with rise in temperature.

The temperature coefficient of resistance is positive for a metals and negative for semiconductors.

Colour code for Carbon resistances; Resistances in series and parallel: Refer to book.

Electrical Energy and Power

A conductor of resistance R, subjected to a potential different 'V' shown in the figure allows a current 'I' to pass through itself.

$\therefore$ Charge transferred in 't' sec $= q = I.t.$

Now Potential difference $V = \frac{\text{Work}}{\text{Charge}} = \frac{W}{q} \Rightarrow W = qV$
$= V.I t.$

This is a measure of the electrical energy consumed in the resistor in time 't'.

$\therefore$ $W = VIt$ or electrical energy consumed $= VIt = I^2Rt$
$= \frac{V^2}{R}t.$

and Electrical power $P = \frac{W}{t} = VI = I^2R = \frac{V^2}{R}$

SI unit of electric energy is joule (= 1 Watt . 1 sec)

The practical unit of electric energy is kW hr with $1\text{ kW hr} = 3.6 \times 10^6\text{ J}$.

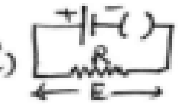
EMF & INTERNAL RESISTANCE

Internal Resistance ; EMF and Potential Difference of a cell

The resistance offered by the electrolyte of a cell during flow of current through itself is called internal resistance(r)

EMF(E)

(i)



(ii)

The potential difference (maximum) across the terminals of a ~~com~~ cell in an open circuit (when no current is drawn) is called E.M.F. [see fig (i)]

The potential difference(V) between the poles of a cell in use when current is being drawn from it is called terminal potential difference.

Relation between E, V and r :- Consider a cell of emf 'E' and internal resistance 'r' used to drive a current 'I' through an external resistance R.

Total resistance of the circuit $= (R + r)$.

$$\therefore \quad E = I(R+r) = IR + Ir = V + Ir$$

$$\Rightarrow \quad r = \frac{E-V}{I} = \left(\frac{E-V}{V/R}\right) = \left(\frac{E-V}{V}\right)R$$

The relations imply that $E > V$; $E = V$ if $I = 0$ or $R = \infty$

Note :- The potential difference V exceeds the EMF of a cell when it is being charged.

Sample Problem 1 :- A wire of resistance 50Ω is drawn out to double its original length. What will be its new (i) resistivity (ii) resistance? Give original resistivity x units.

Ans :- (i) Resistivity does not depend on shape/size. So it still remains x units

(ii) We have $R = \rho \frac{l}{A}$.

So $R' = \rho \frac{l'}{A'}$. For the wire volume remains unchanged

$\therefore lA = l'A' \Rightarrow A' = A \cdot \frac{l}{l'} = A \cdot \frac{l}{2l} = \frac{A}{2}$ $[\because l' = 2l]$

$\therefore R' = \rho \frac{l'}{A'} = \frac{2l\rho}{A/2} = 4\frac{\rho l}{A} = 4R = 4(5\Omega) = 20\Omega$

Sample Problem 2 :- Calculate the conductivity of a material if a 3m long conductor with area of cross-section 0.2 mm^2 has a resistance of 2ohm.

Solution :- We have $R = \rho\frac{l}{A}$ or $\rho = \frac{RA}{l}$.

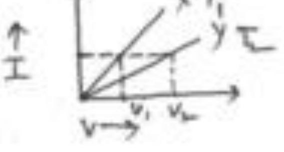
$\therefore$ Conductivity $\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{3}{2 \times 0.2 \times 10^{-6}} = 7.5 \times 10^6 \, Sm^{-1}$

Sample Problem 3 :- Two lamps are marked 60 W; 220 V and 100 W; 220 V. Compare their resistances.

Solution We have $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$.

$\therefore \frac{R_1}{R_2} = \frac{P_2}{P_1}$ ~~or~~ or $\frac{R_{60W}}{R_{100W}} = \frac{100}{60} = \frac{5}{3}$.

Sample Problem 4 :- The V-I graph for a metallic wire at two different temperatures T_1 and T_2 is as shown in Fig 1.77. Which of the two temperatures T_1 and T_2 is higher and why?



Ans. If same current I is to be passed through the wires; X requires a potential difference V_1 and Y requires V_2.

From the graph $V_2 > V_1 \Rightarrow R_Y > R_X$ or $T_2 > T_1$

Sample Problem 5 n identical resistances each of resistance R have resistance X when joined in series and Y when joined in parallel. Calculate X/Y.

Solution :- In series; $X = \underbrace{R + R + \cdots + R}_{(n\ \text{times})} = nR$

In parallel $1/Y = \frac{1}{R} + \frac{1}{R} + \cdots \frac{1}{R}$ (n times) $\Rightarrow \frac{1}{Y} = \frac{n}{R}$ or $Y = \frac{R}{n}$

$\therefore \frac{X}{Y} = \frac{nR}{R/n} = n^2$