CURRENT ELECTRICITY

Electric current :→ It is defined as the rate of flow of charge with time. So current $I = \frac{\text{Charge}}{\text{Time}}$

It is a scalar. Its S I unit is ampere ($= 1 C s^{-1}$).

Average current over a time 't' $= \frac{\text{Total charge transferred } q}{\text{Total time taken } t}$

Instantaneous current $= \frac{dq}{dt}$.

If n electrons are transferred across a section of a conductor a time 't'; then $I = \frac{ne}{t}$

Sample Problem :- The ~~current~~ charge q through a conductor in time 't' is given by $q = at^2 + bt$ [q in coulomb & 't' in second]

(i) What are dimensions of constants 'a' and 'b'?

(ii) What is the value of instantaneous current at $t = 1.5 s$

(iii) Calculate average current between $t = 0 s$ to $t = 4 s$. Given $a = 2; b = 1$ unit

Solution (i) $at^2 = q$ [Dimensionally] $\Rightarrow a = \frac{q}{t^2} = [AT^{-2}]$ and $b = AT^{-1}$

(ii) Current $I = \frac{dq}{dt} = 2at + b \Rightarrow$ At $= t = 1.5 s$; $I = 2(2)(1.5) + 1 = 7 A$.

(iii) $I_{av} = \frac{\text{Total charge}}{\text{Total time}} = \frac{\int_0^4 I\,dt}{4 \text{ sec}} = \frac{|at^2 + bt|_0^4}{4} = \ldots\ldots A$ [Complete the sol.]

Steady Current → The value does not change with time.

Varying Current :- The value changes with time. In the graph 'a' represents steady current and 'b', 'c' (varying).

The Current Carriers :- In Conductors → Free Electrons

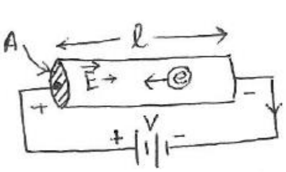
In electrolytes → Positive and Negative ions.

In gases :- Ions and free electrons.

Sample Problem :- Why metals are better conductors than electrolytes?

Ans :- (i) The number of charge carriers (free electron density) per unit volume is very large in conductors than in electrolytes

(ii) Electrons being very light (as compared to ions) drift with very high speeds.



Drift Velocity :→ Metals have free electrons in them. In absence of any external electric field (or potential difference); no current flows in spite of random motion of the electrons. The number of free electrons crossing any section of a conductor from two opposite directions is equal resulting in no net flow across any cross-section of the conductor, ($\therefore I = 0$).

An external

If $\vec{u}_1 ; \vec{u}_2 , \vec{u}_3 , \ldots \vec{u}_n$ denote random thermal velocities of free electrons, then average velocity $\bar{u} = \frac{\vec{u}_1 + \vec{u}_2 + \cdots + \vec{u}_n}{n} = 0$ (zero) . — ①

Suppose a potential difference 'V' is applied across the ends of a conductor of length 'l'; cross-section 'A' resulting in a current I.

Then intensity of electric field = $\vec{E} \left(= \frac{V}{l}\right)$ from positive to negative.

Force $\vec{F}$ on each free electron causes an acceleration.

We have $\vec{F} = -e\vec{E}$ & $\vec{a} = \frac{\vec{F}}{m} = -\frac{e\vec{E}}{m}$ (against $\vec{E}$).

Let $t_1 ; t_2 , \ldots t_n$ denote the time between two successive collisions of free electrons (called free time) ; the velocities attained by free electrons are given by $\vec{V}_1 = \vec{u}_1 + \vec{a}t_1$; $\vec{V}_2 = \vec{u}_2 + \vec{a}t_2 , \ldots \vec{V}_n = \vec{u}_n + \vec{a}t_n$

$\therefore$ Average velocity of free electrons under the action of the field is

$$\bar{V} = \frac{\vec{V}_1 + \vec{V}_2 + \cdots + \vec{V}_n}{n} = \frac{(\vec{u}_1 + \vec{a}t_1) + (\vec{u}_2 + \vec{a}\,t_2) + \cdots (\vec{u}_n + \vec{a}t_n)}{n}$$

$$= \left(\frac{\vec{u}_1 + \vec{u}_2 + \cdots + \vec{u}_n}{n}\right) + \frac{\vec{a}\,(t_1 + t_2 + \cdots + t_n)}{n}$$

$$= |0 + \vec{a}\,\tau| = -\frac{e\vec{E}}{m}\tau \quad [\text{using ①}]$$

Where $\tau = \frac{1}{n}\left(\sum_{i=1}^{n} t_i\right)$ is called mean free time or relaxation time of the electrons.

This velocity $\left(\frac{eE}{m}\tau\right)$ acquired by the free electrons under the action of external field (and in addition to their random thermal velocity) is called drift velocity. This drift is responsible for flow of current.

The value of drift velocity is very small (of the order of 1 mm s^{-1} as compared to the random thermal speed of nearly 10^5 m/s)

Relation between drift velocity and electric current → For the conductor shown ; total volume of conductor = $A.l$

If there are n free electrons per unit volume ; then total number of free electrons in the conductor $= nA.l$.

Total charge on all free electrons in the conductor $= q = (nAl).e$ ①

If v_d denotes the drift speed of free electrons acquired when a potential difference 'V' is applied across a length 'l' of the conductor ; time 't' taken to cross the conductor is given by $t = \frac{l}{v_d}\left(= \frac{l}{\frac{eE}{m}\tau}\right)$. —②

$\therefore$ Current $I = \frac{\text{Charge } q}{\text{Time } t} = \frac{neAl}{l/v_d} = neAv_d$ [Using ① and ②].

Hence $\boxed{I = neAv_d}$ or $v_d = \frac{I}{neA}$. (= drift speed)

or $I = neAv_d = neA\left(\frac{eE}{m}\tau\right) = \frac{e^2nA\tau}{m}E = \frac{e^2nA\tau}{m}\left(\frac{V}{l}\right)$

$\therefore I = \left(\frac{e^2nA\tau}{ml}\right).V$ —③

Electron mobility μ_e is defined as the drift velocity per unit electric field or $\mu_e = \frac{v_d}{E} = \frac{\frac{eE}{m}\tau}{E} = \frac{e\tau}{m}$

$\therefore I = neAv_d = (neA)(\mu_e E) = neA\mu E$.

SI unit of mobility is $m^2V^{-1}s^{-1}$.

Current density (j) is defined as current per unit area

i.e. $j = \frac{I}{A} = nev_d = ne\left(\frac{eE}{m}\tau\right) = \frac{ne^2\tau}{m}E$.

Sample problem :- Two conducting wires X and Y of same diameter but different material are joined in series across a battery. If the number density of free electrons in X is twice that in Y ; find the ratio of drift velocity in the two wires.

Solution $I_x = I_y$ (in series) $\Rightarrow n_1eA_1(v_d)_x = n_2eA_2(v_d)_y$. But $A_1 = A_2$

for $n_1(v_d)_x = n_2(v_d)_y$. Hence $\frac{(V_d)_x}{(V_d)_y} = \frac{n_2}{n_1} = \frac{n_2}{2n_2} = \frac{1}{2}$.